

Tues 3/17

$$10. (1 - \sin^2 \theta)(1 + \tan^2 \theta) + \sin^2 \theta \sec^2 \theta$$

$$= \cos^2 \theta \cdot \sec^2 \theta + \sin^2 \theta \cdot \frac{1}{\cos^2 \theta} = \cos^2 \theta \cdot \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = 1 + \tan^2 \theta = \boxed{\sec^2 \theta}$$

$$11. \cot^2 x (1 + \tan^2 x) - \sin^2 x (1 + \cot^2 x)$$

$$\cot^2 x \cdot \sec^2 x - \sin^2 x \cdot \csc^2 x = \frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} - \sin^2 x \cdot \frac{1}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

$$= \csc^2 x - 1 = \boxed{\cot^2 x}$$

$$12. \sin^2 x (\csc^2 x - 1) + \cos^2 x (\sec^2 x - 1)$$

$$\sin^2 x \cdot \cot^2 x + \cos^2 x \cdot \tan^2 x = \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} + \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x} = \cos^2 x + \sin^2 x = \boxed{1}$$

$$13. \frac{\tan \theta + \sec \theta}{1 + \sin \theta} = \frac{\left( \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right)}{(1 + \sin \theta)} \cdot \frac{\cos \theta}{\cos \theta} = \frac{\sin \theta + 1}{(1 + \sin \theta) \cos \theta} = \frac{1}{\cos \theta} = \boxed{\sec \theta}$$

to bust out the fractions

$$14. \frac{\tan x + \cot x}{\sec x \csc x} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} = \frac{\left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)}{\left( \frac{1}{\cos x \sin x} \right)} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{1} = \boxed{1}$$

$$15. \frac{\sin A + \tan A}{\cot A + \csc A} = \frac{\left( \frac{\sin A}{\cos A} + \frac{\sin A}{\cos A} \right)}{\left( \frac{\cos A}{\sin A} + \frac{1}{\sin A} \right)} \cdot \frac{\sin A \cos A}{\sin A \cos A} = \frac{\sin^2 A \cos A + \sin^2 A}{\cos^2 A + \cos A}$$

$$= \frac{\sin^2 A (\cos A + 1)}{\cos A (\cos A + 1)} = \frac{\sin^2 A}{\cos A} = \sin A \cdot \frac{\sin A}{\cos A} = \boxed{\sin A \tan A}$$

$$16. \frac{\sin x - \cos x}{\tan x \csc x - \sec x \cot x} = \frac{\sin x - \cos x}{\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} - \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}} = \left( \frac{\sin x - \cos x}{\frac{1}{\cos x} - \frac{1}{\sin x}} \right) \cdot \frac{\cos x \sin x}{\cos x \sin x}$$

$$= \frac{\cos x \sin x (\sin x - \cos x)}{\sin x - \cos x} = \boxed{\cos x \sin x}$$

$$17. \frac{\sin x}{\cos x \tan x} + \frac{\cos x}{\sin x \tan x} = \frac{\sin x}{\cos x \cdot \frac{\sin x}{\cos x}} + \frac{\cos x}{\sin x \cdot \frac{\sin x}{\cos x}} = \frac{\sin x}{\sin x} + \frac{\cos x}{\frac{\sin^2 x}{\cos x}}$$

$$= 1 + \frac{\cos x \cdot \cos x}{\sin^2 x} = 1 + \frac{\cos^2 x}{\sin^2 x} = 1 + \cot^2 x = \boxed{\csc^2 x}$$